PARABOLA

EXERCISE – I HINTS & SOLUTIONS

Sol.1 C

a = $\perp^r$ distance from (3, 4) to the tangent at vertex

$= \left| \dfrac{3+4-7-5\sqrt{2}}{\sqrt{2}} \right|$

a = 5

LR = 4a = 20

Sol.2 C

Directrix : x + y – 2 = 0

Focus to directrix distance = 2a

$2a = \left| \dfrac{0+0-2}{\sqrt{2}} \right|$

$2a = \sqrt{2}$

$LR = 4a = 2\sqrt{2}$

Sol.3 B

(A) $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$ Ellipse

(B) $x^2 - 2 = -\left(2\cos^2\dfrac{t}{2} - 1\right)$

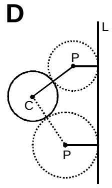
$x^2 - 2 = -\left(\dfrac{y}{2} - 1\right)$

$x^2 - 2 = -\dfrac{y}{2} + 1$

$x^2 = -\dfrac{y}{2} + 3$

$x^2 = -\dfrac{1}{2}(y - 6)$ Parabola

Sol.4 D



Locus of P will be parabola

Sol.5 A

$y^2 = 4x$ a = 1

$P(t^2, 2t)$ $t_1 t_2 = -1$ For focal chord

$t_2 = -\dfrac{1}{t}$

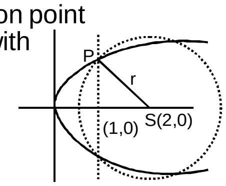
$Q\left(\dfrac{1}{t^2}, \dfrac{-2}{t}\right)$

$PQ = \sqrt{\left(t^2 - \dfrac{1}{t^2}\right)^2 + \left(2t + \dfrac{2}{t}\right)^2}$

$= \left(t + \dfrac{1}{t}\right)\sqrt{\left(t - \dfrac{1}{t}\right)^2 + 4} = \left(t + \dfrac{1}{t}\right)^2$

Sol.6 B

$P(1, 2\sqrt{2})$ Intersection point of x = 1 with $y^2 = 8x$



$r^2 = SP^2$

$= (1-2)^2 + (2\sqrt{2})^2$

= 1 + 8 = 9

equation of circle as centre (2, 0) ; r = 3

$(x - 2) + y^2 = 9$

Sol.7 B

$x = t^2 + 1$; $y = 2t$ $\Rightarrow t = \dfrac{y}{2}$

$x = \dfrac{y^2}{4} + 1$ ....(i)

$x = 2s$; $y = \dfrac{2}{s}$ $\Rightarrow s = \dfrac{2}{y}$

$x = \dfrac{4}{y}$

$\dfrac{4}{y} = \dfrac{y^2}{4} + 1$

$y^3 + 4y - 16 = 0 \Rightarrow \left.\begin{matrix} y = 2 \\ x = 2 \end{matrix}\right\}$ POI

Aliter

Assume a point on hyperbola $\left(2t, \dfrac{2}{t}\right)$

Put in parabola

$2t = \dfrac{1}{t^2} + 1$

$2t^3 - t^2 - 1 = 0$

t = 1 will satisfy point (2, 2)

Sol.8 D

PM = SM

$PM^2 = SM^2$

$(a + at^2)^2 = 4a^2 + 4a^2t^2$

$a^2 + a^2t^4 + 2a^2t^2 = 4a^2 + 4a^2t^2$

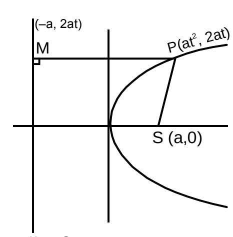
$1 + t^4 + 2t^2 = 4 + 4t^2$

$t^4 - 2t^2 - 3 = 0$

$t^2 = 3$, $t^2 = -1$

$SP = a + at^2$

$= a + 3a = 4a$



Sol.9 C

$M_{OP} = \dfrac{2}{t_1}$

$M_{OQ} = \dfrac{2}{t_2}$

$\dfrac{2}{t_1} \times \dfrac{2}{t_2} = -1 \Rightarrow t_1 t_2 = -4$ ......(i)

Equation of PQ

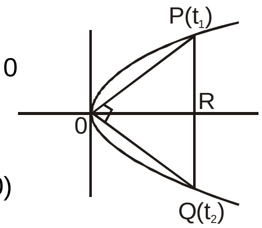
$2x - (t_1 + t_2)y + 2at_1t_2 = 0$

put $y = 0$

$x = -at_1t_2$

$x = -at_1\,t_2$

$x = 4a$ $\qquad$ $R(4a, 0)$



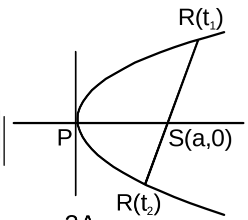
Sol.10 C

$$A = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left|\left[2a^2t_1^2t_2 - 2a^2t_1t_2^2\right]\right|$$

$$= \frac{1}{2}\left|at_1\ t_2\left[2at_1 - 2at_2\right]\right| \quad \frac{2A}{a} = |y_1 - y_2|$$

$t_1t_2 = -1$



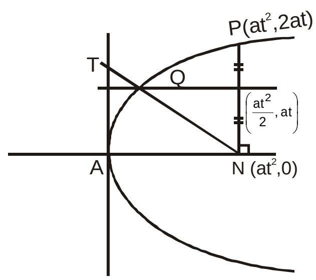
Sol.11 B

$N(at^2, 0)$

solve $y = at$ with curve $y^2 = 4ax$

$$x = \frac{at^2}{4}$$

$$Q\left(\frac{at^2}{4}, at\right)$$



Equation of QN $\quad y = \dfrac{dt}{\left(\dfrac{at^2}{4} - at^2\right)}(x - at^2)$

put $x = 0$ $\quad y = \dfrac{4}{3}at$

$T\left(0, \dfrac{4}{3}at\right)$ $\quad AT = \dfrac{4}{3}at$

$PN = 2at$

$$\frac{AT}{PN} = \frac{4/3\ at}{2at} = \frac{2}{3} \quad \text{so } k = \frac{2}{3}$$

Sol.12 D

$y^2 = x - c$; $a = 1/4$

Slope of tangent $= \dfrac{1}{t}$

so $\dfrac{1}{t_1\,t_2} = -1$

$t_1t_2 = -1$ $\qquad$ ....(i)

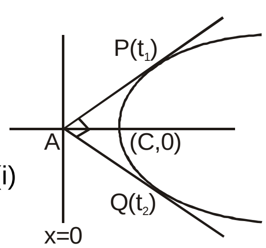
$A(at_1t_2 + C, a(t_1 + t_2))$

$at_1t_2 + C = 0$

$C = -at_1t_2$

$C = a$

$C = \dfrac{1}{4}$

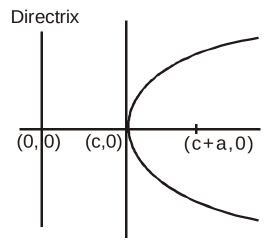


Aliter

$$\frac{c + a + 0}{2} = c$$

$c + a = 2c \Rightarrow c = a$

$\Rightarrow c = 1/4$



Sol.13 C

$y^2 = 4ax$

Slope $= \dfrac{1}{t}$

$\dfrac{1}{t_1} = \dfrac{2}{t_2}$

$\Rightarrow t_2 = 2t_1$ ......(1)

$R[at_1t_2, a(t_1 + t_2)]$

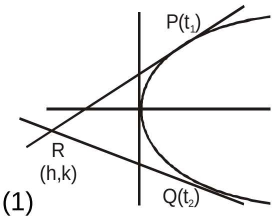
$h = at_1\,t_2$, $k = a(t_1 + t_2)$

$k = 3at_1 \Rightarrow t_1 = \dfrac{k}{3a}$

$h = 2at_1^2$

$h = 2a\dfrac{k^2}{9a^2} \Rightarrow k^2 = \dfrac{9}{2}ah$

$y^2 = \dfrac{9}{2}ax$



Sol.14 C

Δ PUT $\cong$ Δ PLT

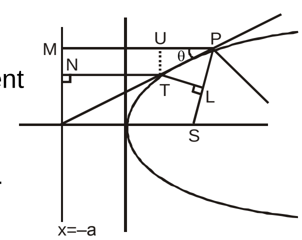
Both Δ are congurrent

Hence PU = PL

PM = SP

PM – PL = SP – PL

TN = MU = SL



Sol.15 D

$(x - 1)^2 = 8y$; $a = 2$ $\quad x - 1 = 0, y = 2$

$x^2 = 8y$; $x = 1, y = 2$

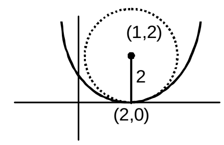
vertex (1, 0)

Focus (1, 2)

Radius of circle = 2

$(x - 1)^2 + (y - 2)^2 = 4$

$x^2 + y^2 - 2x - 4y + 1 = 0$



Sol.16 B

Normal at $P(at_1^2, 2at_1)$

$a = 1$

$P(t_1^2, 2t_1)$

$y + t_1x = 2t_1 + t_1^3$ .....(1)

slope $= 1 = -t_1$

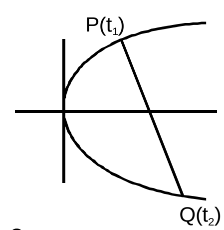
$t_1 = -1$

$P(1, -2)$ $\qquad$ $t_2 = -t_1 - \dfrac{2}{t_1}$

$Q(t_2^2, 2t_2)$ $\qquad$ $t_2 = 1 + 2 = 3$

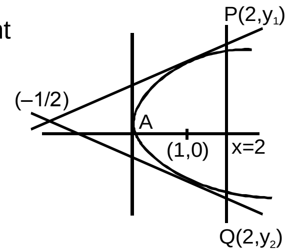
$Q(9, 6)$

$PQ = \sqrt{(9-1)^2 + (6+2)^2} = 8\sqrt{2}$



Sol.17 B

Equation of tangent



$y = mx + \frac{a}{m}$

$a = 1$

$(-1, 2)$

$2 = -m + \frac{1}{m}$

$m^2 + 2m - 1 = 0 \begin{cases} m_1 \\ m_2 \end{cases}$

$m_1 + m_2 = -2$

$m_1 m_2 = -1$

Length PQ $= |y_1 - y_2|$

$= |m_1x + 1/m_1 - m_2x - 1/m_2|$ at $x = -2$

$= \left|2(m_1 - m_2) - \left(\frac{m_1 - m_2}{m_1 m_2}\right)\right|$

$= |3(m_1 - m_2)| = 3\left(2\sqrt{2}\right) = 6\sqrt{2}$

Sol.18 D

$y^2 + 4y - 6x - 2 = 0$

$y^2 + 4y + 4 - 6x - 6 = 0$; $a = \frac{3}{2}$

$(y + 2)^2 = 6(x + 1)$

$Y^2 = 6X$ vertex $(-1, -2)$

POI of tangents $t_1 t_2 = -1$

$[at_1t_2, a(t_1 + t_2)]$

$h + 1 = at_1t_2$

$h + 1 = -\frac{3}{2}$

$2h + 2 = -3$

$2h + 5 = 0 \Rightarrow 2x + 5 = 0$

Sol.19 C

Let point $P(x_1, y_1)$

$x_1 - y_1 + 3 = 0$

C.O.C. w.r.t. (x_1, y_1) of $y^2 = 4ax$

$yy_1 = 4(x + x_1)$

$y(x_1 + 3) = 4x + 4x_1$

$yx_1 + 3y - 4x - 4x_1 = 0$

$(3y - 4x) + x_1(y - 4) = 0$

$L_1 + \lambda L_2 = 0$

$L_1 = 0$ & $L_2 = 0$

$3y = 4x$ $y = 4$

$x = 3$

point $(3, 4)$

Sol.20 C

Let : $4x - 7y + 10 = 0$ ....(1)

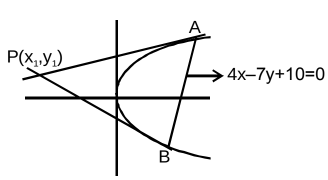
C.O.C. w.r.t. $P(x_1, y_1)$

$yy_1 = 2a(x + x_1)$; $a = 1$

$yy_1 = 2(x + x_1)$

$2x - yy_1 + 2x_1 = 0$ ....(2)

(1) & (2) are same



$\frac{4}{2} = \frac{-7}{-y_1} = \frac{10}{2x_1}$

$x_1 = \frac{5}{2}$; $y_1 = \frac{7}{2}$ POI $\left(\frac{5}{2}, \frac{7}{2}\right)$

Sol.21 A

$y^2 = 8x$; $a = 2$

Area $= \frac{(y_1^2 - 8x_1)^{3/2}}{4}$; $(4, 6)$

$= \frac{(36 - 32)^{3/2}}{4} = \frac{8}{4} = 2$ sq. units

Sol.22 C

Equation of PQ

$(t_1 + t_2)y = 2x + 2at_1t_2$

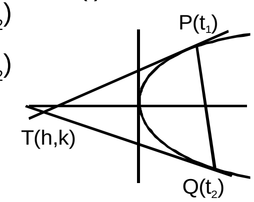
passes through $(-a, b)$

$b(t_1 + t_2) = -2a + 2at_1t_2$ ....(i)

$h = at_1t_2$ & $k = a(t_1 + t_2)$

POI of tangents

$h = at_1t_2$ & $k = a(t_1 + t_2)$



$\frac{bk}{a} = -2a + 2h$

$bk = -2a^2 + 2ah$

$by = -2a^2 + 2ax$

$by = 2a(x - a)$

Sol.23 C

Tangent at P of $y^2 = 4ax$

$yy_1 = 2a(x + x_1)$ .....(1)

Let Mid point (h, k)

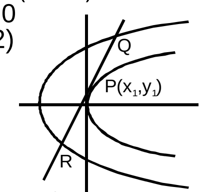
$T = S_1$

$yk - 2a(x + h) - 4ab = k^2 - 4a(h + b)$

$yk - 2ax - 2ah + 4ah - k^2 = 0$

$yk - 2ax + 2ah - k^2 = 0$ ....(2)

(1) & (2) are same



$\frac{k}{y_1} = \frac{-2a}{-2a} = \frac{2ah - k^2}{-2ax_1}$

$k = y_1$; $-2ax_1 = 2ah - k^2$

$-2ax_1 = 2ah - y_1^2$; $y_1^2 = 4ax_1$

Mid point $-2ax_1 = 2ah - 4ax_1$

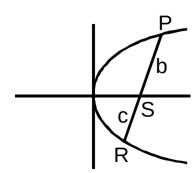
(x_1, y_1) $2ah = 2ax_1$

$h = x_1$

Sol.24 A

$y^2 = 8x$

$SP = 6$



$\frac{1}{b} + \frac{1}{c} = \frac{1}{a}$

$\frac{1}{c} = \frac{1}{a} - \frac{1}{b}$

$c = \frac{ab}{b - a}$ $b = 6, a = 2$

$= \frac{12}{4} = 3$

Sol.25 C

$y^2 = 4a(x = \ell_1)$ $x^2 = 4a(y - \ell_2)$

let the POC (h, k)

$2yy' = 4a$ $2x = 4ay'$

$y' = \left.\frac{2a}{y}\right|_{(h,k)} = \frac{2a}{k}$ ...(1) $y' = \left.\frac{x}{2a}\right|_{(h,k)}$

(1) and (2) are equal $= \frac{h}{2a}$ ....(2)

$\frac{2a}{k} = \frac{h}{2a}$

$hk = 4a^2$

$xy = 4a^2$